Fractured Dimension
By David Silver
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I

"Today we are going to explore fractal geometry" announced Dr Jeremy Blandford to his first year undergraduate class. "These are shapes that are comprised of re-iterations of themselves. Take the overall shape of a fir tree for instance. Now examine a branch. It looks like a fir tree. Examine a sub-branch. It looks like a fir tree. Examine a frond. It looks like a fir tree. Nature, in all its cleverness, has used the same design formula over and over again to build a biological object ranging from its overall structure almost right down to the cellular level. The object is made up of copies of itself. This is a fractal.

"Ditto for a coastline. Look at Britain from high up in an aeroplane or even from space … if you can afford the fare on a student's allowance of course". Polite laughter. "It's crinkled. Stand on the beach and look at the cliffs. They are crinkled with nooks, crannies and caves. Enter a cave. Its walls will be crinkled. Shrink yourself down to the size of an ant and enter one of those crinkles. There are more crinkles. Shrink yourself down to the size of a microbe and enter one of the crinkles inside a crinkle. There are more crinkles, all the way down to the molecular level. And that is where the limits are – at the molecular level. Or is it?" He paused for effect, waiting for the usual outbreak of smutty murmurs about crinkles and wrinkles to subside.

"There are many mathematical formulae that can generate a shape that is fractal in construction" he continued. "You take a number and feed it into the formula, which calculates a new number. You plot this number pair on the x-y axes of a piece of graph paper. Then you take the new number and feed it into the same formula, obtaining another new number. You plot this new pair on the graph, and so on. Eventually you end up with a shape of some sort. Adrien Douady was a 20th century mathematician who discovered a formula he named 'The Mandelbrot Set' in tribute to his mathematical peer Benoit Mandelbrot who actually discovered fractal geometry".

He advanced his slide show to display the formula.

The **Mandelbrot set** is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge when iterated from $z = 0$, i.e., for which the sequence $f_c(0)$, $f_c(f_c(0))$, etc., remains bounded in absolute value.

"It rather trips off the tongue doesn't it? And for those of you who don't know, a complex number is one which includes the square root of minus 1, an imaginary number. And I don't want any wisecracks in a week's time that you'd imagined you'd completed your assignments on time. Actually this formula is quite simple and boils down to result 2 equals result 1 squared plus 2, result 3 equals result 2 squared plus 3 and so on. And when the results of this formula are used to generate a shape either on graph paper or inside a computer, a remarkable formation appears". He again advanced the slide show to display the image in question, large conjoined blobs with wiry filaments and crenellated extrusions all around their perimeters.
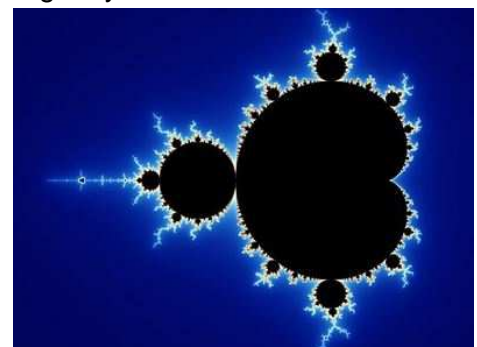
He didn't mention that the formation had been nicknamed by some as The Snowman due to the fact that it seemed vaguely humanoid, resembling a stout but rather sinister snowman lying on its side. "The magical thing about this construct" he continued "is that if you were to use your computer to zoom into one of those crenellations … it's a bit difficult doing this on a piece of graph paper ha ha … your screen fills up with the magnified section and looks very much like the original".

Yes, you would again see a stout but rather sinister snowman lying on its side with wiry extensions and crenellated extrusions all around its perimeter. You could zoom into one of the new crenellations and again your screen would fill up with the magnified section which would look like … yes, a stout but rather sinister snowman lying on its side with wiry extensions and crenellated extrusions all around its perimeter.

He continued. "You can in theory repeat this process an infinite number of times, drilling down into an endless imaginary world. Some say you are peering into the foundations of the Universe. Some say you are viewing the depths of Hell itself". A cynical titter rippled through the class. But the class wasn't aware that that was what Blandford actually believed. A leading expert in computational mathematics and also a devoutly religious man, he had become obsessed with the idea that fractals held the key to how God Himself had designed Heaven, Earth, Hades and the interfaces between them. He belonged to a small and secretive group of like-minded thinkers who had named themselves The Mandelbrot Sect. Outsiders who are aware that the Mandelbrot Set and similar fractal constructs are neither two-dimensional nor three dimensional, but somewhere in between, 2.63 for example, might tend toward some sympathy with the Sect's view. Certainly the fact that an object with 2.63 dimensions can exist can instil creeping unease in even the sanest mind, sowing doubts about the apparent solidity and rationality of the universe.

Blandford had equipped his study with one of the most powerful scientific computers he could acquire with which to research his sect's theories. He would spend hours losing himself trying to establish the boundary between order and chaos. For some formulae, you might enter an initial number of, say 0.7 and receive a result of 0.54. You enter 0.54 and receive a result of 0.8. 0.8 begets 0.6, 0.6 gives 0.79 and so on. The numbers all cycle round a reasonably consistent range. Yet tweak the starting condition by just a minute amount, say to 0.7001 and the results might frustratingly become 197 then minus 5976543 then 453298746352 and so on as they spiral outward toward infinity, demanding a piece of graph paper larger than the entire known universe. Clearly in this case the formula has gone wild. Chaotic in fact. Just like the starting conditions for our Universe, according to the Sect, and actually something widely considered and discussed in scientific circles and amateur philosophy groups around the world. If one of those conditions had been slightly different - the strength of gravity for instance - we wouldn't be here now.

The Mandelbrot Sect believed that, by delving deeply enough into the nano-mathematical world of the Set, always following the Right Path to avoid the ghastly Region Of Chaos, the true construction of the universe would be revealed to the viewer, who would find him or her self sitting at the Left Hand of Satan and, more importantly, at the Right Hand of God as His Own Mathematician.

II

Dr Blandford lived alone. His wife had long since left him, citing as the reasons neglect, his serial infidelity with books of graph paper and computer programming languages and his involvement with a lunatic quasi-religious sect. Their daughter Laura was a perpetual Humanities student at a London university and Blandford would see her only every few weeks when she would drop in unannounced.

It was on the afternoon of an especially wintery New Year's Eve that Blandford achieved his greatest, and final, breakthrough. Having been hampered in his research up to now by the limitations of the smallest number his computer, operating system and programming languages could accommodate, he had devised an ingenious method to extend this limit by orders of magnitude whilst not exceeding the ability of the computer to deliver results, not just within the duration of his own lifetime but actually within seconds. He had performed a number of preliminary tests to validate his techno-mathematical breakthrough and was ready to embark on what he anticipated to be a voyage to ultimate knowledge.

He ran the command to generate the initial Mandelbrot Set. The sinister Snowman appeared. In his mind it appeared to be almost sentient, challenging and mocking him.
He highlighted a small area around a crenellation at the bottom right of the image and commanded magnification. The area expanded by a factor of around 400, driving away The Snowman and presenting him with … The Snowman. A slightly more mocking version he thought. He repeated the exercise, becoming a little disquieted by the apparently evolving expression of the object. But he was determined and, now the ship was edging out of the harbour as it were, he was going to engage the engines at full speed. He'd instructed the computer as to which co-ordinates of the shape should be iteratively explored and now he clicked on the 'Automate Repetition' command.

III

Laura let herself in through the front door, carrying a small valise and a bag of belated Christmas knick-knacks for her father. She called out for him but there was no answer. She assumed he was in because the intruder alarm had not sounded and he was a stickler for setting it whenever he left the house.

She placed her baggage in the kitchen and went to find him. Sure enough, there he was in his study, sitting bolt upright and motionless in front of his computer, the screen flickering in an alarming manner. She spoke again but he showed no sign of awareness, eyes fixed on the screen, one white-knuckled hand gripping the edge of the desk and the other clutching the computer's mouse.

She went round to view his face, calling 'Dad, Dad'. His eyes were wide open staring at the screen. Seeing him so unresponsive she feared he was having a seizure induced by the flickering patterns. She instinctively ran to the wall and pulled the power cord out of its socket. Then she screamed. He, the chair he sat on, the desk and the computer started to fade slowly from sight, leaving a sort of shrinking spherical shadow, a set of severed computer cables and a circular depression in the floor.

In the course of the subsequent investigations into Blandford's disappearance the remnant of the shadow was discovered to be visible only as an artefact out of the corner of one's eye. Direct vision could not seem to perceive it. In years to come the house, and in generations to come when the house no longer existed, the land itself, would come to be considered haunted, and frightening folk tales would be invented.

IV

He was falling through the sub-nanoscopic universe, The Snowman appearing, mocking, disappearing, only for another figure of ever-increasing mockery to take its place. He was terrified and exhilarated as he plunged ever downward, or perhaps inward. The computer magnified and zoomed, below the Planck length and beyond, forever magnifying, zooming, magnifying. His physical self, the computer, his formulae, guided by his intellect, travelled into the world of the infinitesimal, forever seeking the Left Hand of Satan and the Right Hand of God.

THE END